CHAPTER (4)

SOLVED PROBLEMS

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1.a **Uniform Flow** $\frac{\partial V}{\partial s} = 0$ **Non-Uniform Flow** $\frac{\partial V}{\partial s} \neq 0$

1.b **Steady Flow** $\frac{\partial V}{dt} = 0$ **Non-Steady Flow** $\frac{\partial V}{dt} \neq 0$

2.a **Acceleration:** $a = a_t + a_n$ $a_t = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t}\right)$ $a_n = \left(\frac{V^2}{r}\right)$

2.b **Acceleration of Liquid in a Tank:** $\frac{dz}{dl} = \frac{a_l}{g} = \frac{a_x \cos \alpha}{g}$

3. **Eulerian Approach:**

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \qquad a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \qquad a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

4. **Euler Equation of motion:** $-\frac{d}{dl}(p + \gamma z) = \rho a_l$

5. **Bernoulli Equation:** $\left(p + \gamma z + \rho\frac{V^2}{2}\right) = C$ $OR \quad \frac{p}{\gamma} + z + \frac{V^2}{2g} = C$

6. **Pitot Tube Equation** $V_2 = \left[\frac{2}{\rho}\left(p_{z,1} - p_{z,2}\right)\right]^{1/2}$

6. **Pressure Coefficient** $C_p = \frac{p - p_0}{\frac{1}{2}\rho V_0^2}$ $C_p = \frac{h - h_0}{\frac{1}{2}\rho V_0^2}$

7. The rotational velocity about x, y and z axes

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \qquad \Omega_z = \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right)$$

8. Vorticity $(\omega) = 2\Omega$

9. Free Vortex Flow $V = \frac{C}{r}$

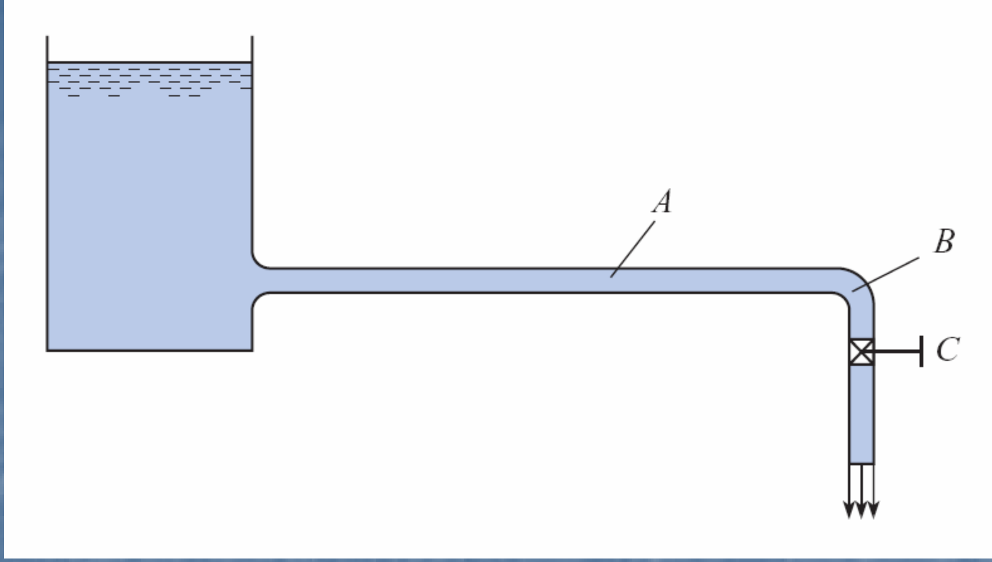
10. Forced Vortex Flow $V = \omega r$

11. Pressure Variation in Rotating Flow $\left(\frac{p}{\gamma} + z - \frac{\omega^2 r^2}{2g}\right) = C$

12. The Pressure Difference between point (1); i.e. the center of the tornado and the outer edge of the tornado

$$p_1 - p_0 = -\rho V_{max}^2$$

Problem 4.1 (p. 126)



PROBLEM 4.1

Situation: The valve in a system is gradually opened to have a constant rate of increase in discharge.

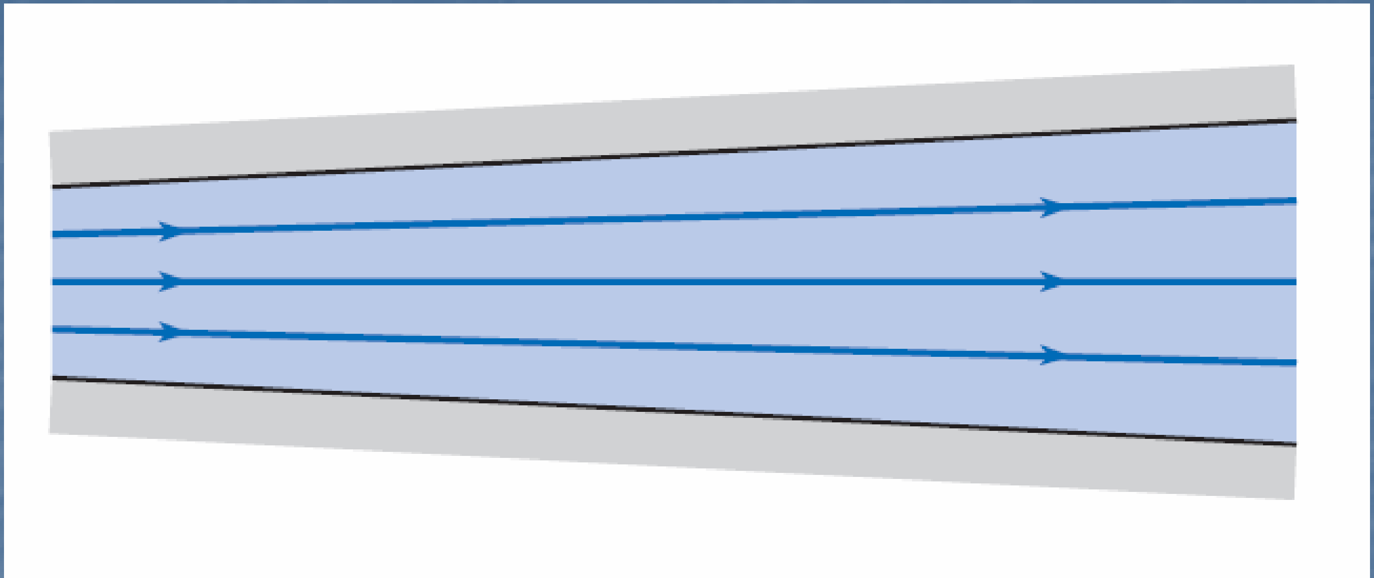
Find: Describe the flow at points A and B.

ANALYSIS

B: Non-uniform, unsteady.

A: Unsteady, uniform.

Problem 4.2 (p. 126)



PROBLEM 4.2

Situation: Water flows in a passage with flow rate decreasing with time.

Find: Describe the flow.

ANALYSIS

(b) Unsteady and (d) non-uniform.

(a) Local and (b) convective acceleration.

$$\frac{\partial V}{dt} \neq 0 \qquad \frac{\partial V}{\partial s} \neq 0$$

$$a_t = \left(V \frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

Problem 4.18 (p. 128)

PROBLEM 4.18

Situation: A flow with this velocity field: $u = xt + 2y,\ v = xt^2 - yt,\ w = 0.$

Find: Acceleration, a, at location (1,2) and time $t = 3$ seconds.

ANALYSIS

Acceleration in the x-direction

$$\begin{aligned} a_x &= u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial z + \partial u/\partial t \\ &= (xt + 2y)(t) + (xt^2 - yt)(2) + 0 + x \end{aligned}$$

At $x = 1$ m, $y = 2$ m and $t = 3$ s

$$a_x = (3+4)(3) + (9-6)(2) + 1 = 21 + 6 + 1 = 28 \text{ m/s}^2$$

Acceleration in the y-direction

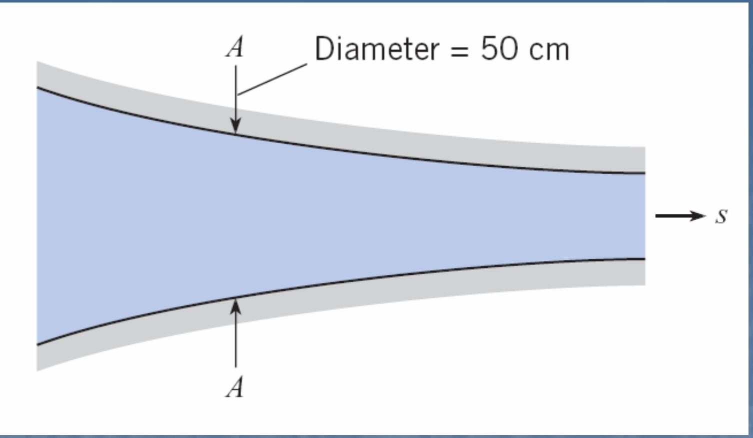
$$\begin{aligned} a_y &= u\partial v/\partial x + v\partial v/\partial y + w\partial v/\partial z + \partial v/\partial t \\ &= (xt + 2y)(t^2) + (xt^2 - yt)(-t) + 0 + (2xt - y) \end{aligned}$$

At $x = 1$ m, $y = 2$ m and $t = 3$ s

$$a_y = (3+4)(9) + (9-6)(-3) + (6-2) = 63 - 9 + 4 = 58 \text{ m/s}^2$$

$$\boxed{\mathbf{a} = 28\,\mathbf{i} + 58\,\mathbf{j} \text{ m/s}^2}$$

Problem 4.21 (p. 128)



$Given: Q_0 = 0.985\, m^3/s,\; Q_1 = 0.5\, m^3/s,\; t_0 = 1\, s$

$\partial V/\partial s = 2\, m/s$ $d = 50\, cm$

Situation: Flow occurs in a tapered passage. The velocity is given as

$$V = Q/A$$

and

$$Q = Q_o - Q_1 \frac{t}{t_o}$$

The point of interest is section AA, where the diameter is 50 cm. The time of interest is 0.5 s.

Find: (a) Velocity at section AA: V
(b) Local acceleration at section AA: a_ℓ $\frac{dV}{dt}$
(c) Convective acceleration at section AA: a_c $V\frac{dV}{ds}$

Problem 4.21 (p. 128)

$Given: Q_0 = 0.985\, m^3/s,\ Q_1 = 0.5\, m^3/s,\ t_0 = 1\, s$

$\partial V/\partial s = 2\, m/s$

$d = 50\, cm$

$$
\begin{aligned}
Q &= Q_0 - Q_1 t/t_0 = 0.985 - 0.5t \quad \text{(given)} \\
V &= Q/A \quad \text{(given)} \\
\frac{\partial V}{\partial s} &= +2\ \frac{\text{m}}{\text{s}}\ \text{per m} \quad \text{(given)}
\end{aligned}
$$

The velocity is

$t = 0.5\, s$

$$
\begin{aligned}
V &= Q/A \\
&= (0.985 - 0.5 \times 0.5)/(\pi/4 \times 0.5^2)
\end{aligned}
$$

$$\boxed{V = 3.743\ \text{m/s}}$$

Local acceleration

$t = 0.5\, s$

$$
\begin{aligned}
a_\ell &= \partial V/\partial t = \partial/\partial t(Q/A) \\
&= \partial/\partial t((0.985 - 0.5t)/(\pi/4 \times 0.5^2)) \\
&= -0.5/(\pi/4 \times 0.5^2)
\end{aligned}
$$

$$\boxed{a_\ell = -2.55\ \text{m/s}^2}$$

Convective acceleration

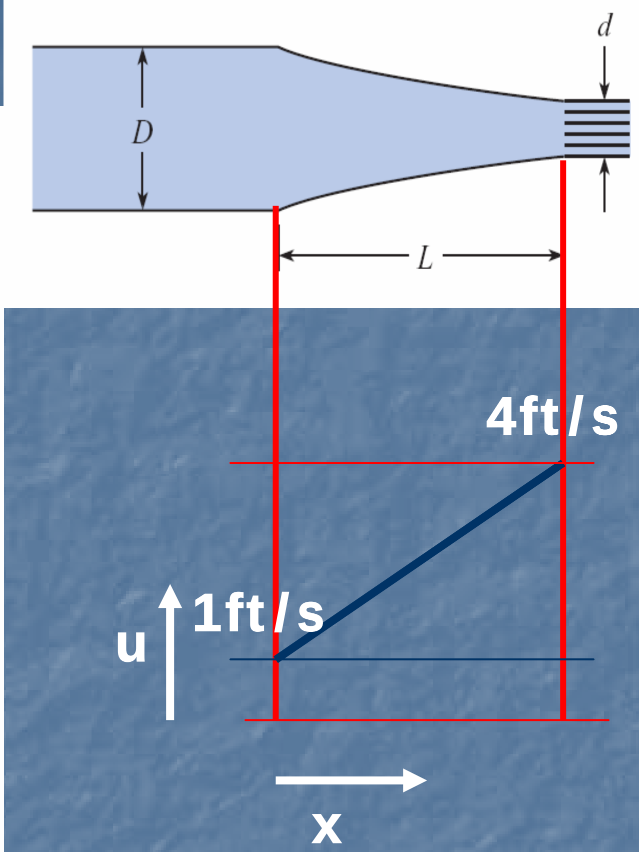
$t = 0.5\, s$

$$
\begin{aligned}
a_c &= V \partial V/\partial s \\
&= 3.743 \times 2
\end{aligned}
$$

$$\boxed{a_c = +7.49\ \text{m/s}^2}$$

Problem 4.22 (p. 128)

$$a_t = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$



PROBLEM 4.22

Situation: One-dimensional flow occurs in a nozzle. Velocity varies linearly from 1 ft/s at the base to 4 ft/s at the tip. The nozzle is 18 inches long.

Find: (a) Convective acceleration: a_c **At mid-point**
(b) Local acceleration: a_ℓ

ANALYSIS

Velocity gradient

$$\begin{aligned} dV/ds &= (V_{\text{tip}} - V_{\text{base}})/L \\ &= (4-1)/1.5 \\ &= 2\ \text{s}^{-1} \end{aligned}$$

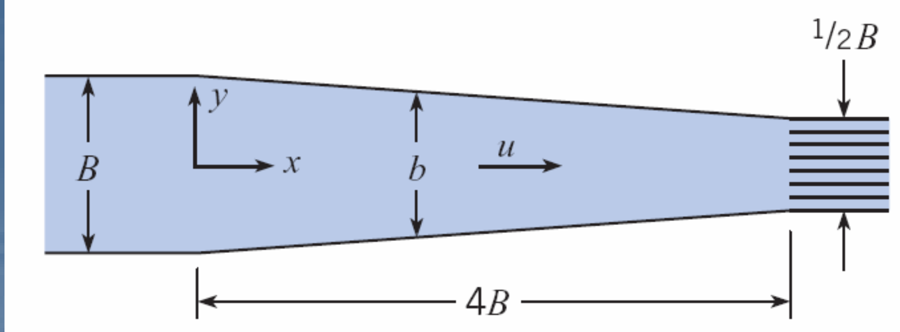
Acceleration at mid-point

$$\begin{aligned} V &= (1+4)/2 \\ &= 2.5\ \text{ft/s} \\ a_c &= V\frac{dV}{ds} \\ &= 2.5 \times 2 \\ & \boxed{a_c = 5\ \text{ft/s}^2} \end{aligned}$$

Local acceleration

$$\boxed{a_\ell = 0}$$

Problem 4.24 (p. 128)



PROBLEM 4.24

Situation: Flow in a two-dimensional slot with

$$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right)$$

Find: An expression for local acceleration midway in nozzle: a_l and $y = 0$

ANALYSIS

$$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right) \text{ but } b = B/2$$

$$V = \left(\frac{4q_o}{B}\right)\left(\frac{t}{t_o}\right)$$

$$a_l = \partial V/\partial t$$

$$\boxed{a_l = 4q_o/(Bt_o)}$$

Problem 4.27 (p. 129)

Euler Equation

$$-\frac{d}{dl}(p+\gamma z)=\rho a_l$$

PROBLEM 4.27

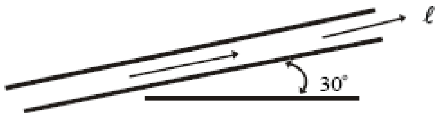
Situation: Flow through an inclined pipe at 30° from horizontal and decelerating at 0.3g.

Find: Pressure gradient in flow direction.

APPROACH

Apply Euler's equation.

ANALYSIS

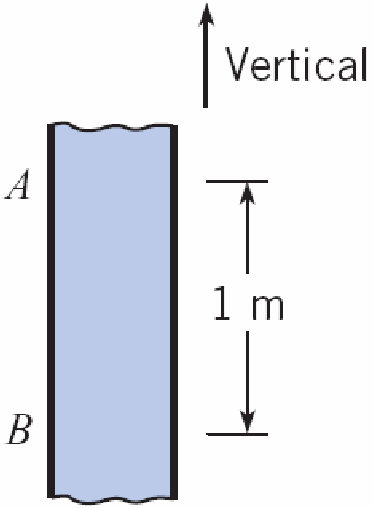


Euler's equation

$$\begin{aligned}
\partial/\partial\ell(p+\gamma z) &= -\rho a_\ell \\
\partial p/\partial\ell+\gamma\partial z/\partial\ell &= -\rho a_\ell \\
\partial p/\partial\ell &= -\rho a_\ell-\gamma\partial z/\partial\ell \\
&= -(\gamma/g)\times(-0.30g)-\gamma\sin 30^\circ \\
&= \gamma(0.30-0.50)
\end{aligned}$$

$$\boxed{\partial p/\partial\ell=-0.20\gamma}$$

Problem 4.27 (p. 129)



PROBLEM 4.29

Situation: A hypothetical liquid with zero viscosity and specific weight of 10 kN/m^3 flows through a vertical tube. Pressure difference is 12 kPa.

Find: Direction of acceleration.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$\rho a_\ell = -\partial/\partial\ell(p+\gamma z)$$
$$a_\ell = (1/\rho)(-\partial p/\partial\ell - \gamma\partial z/\partial\ell)$$

Let ℓ be positive upward. Then $\partial z/\partial\ell = +1$ and $\partial p/\partial\ell = (p_A - p_B)/1 = -12{,}000$ Pa/m. Thus

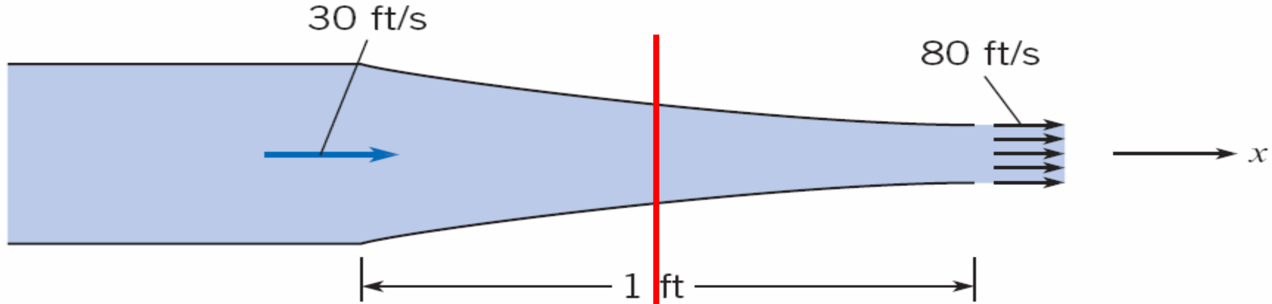
$$a_\ell = (g/\gamma)(12{,}000 - \gamma)$$
$$a_\ell = g((12{,}000/\gamma) - 1)$$
$$a_\ell = g(1.2 - 1.0) \text{ m/s}^2$$

a_ℓ has a positive value; therefore, acceleration is upward. Correct answer is a).

Problem 4.36 (p. 129)

$$a_t = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

Find pressure gradient Half-way through the nozzle



$$\frac{\partial p}{\partial x} = ?$$

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$d/dx(p + \gamma z) = -\rho a_x$$

but z =const.; therefore

$$dp/dx = -\rho a_x$$
$$a_x = a_{\text{convective}} = V dV/dx$$
$$dV/dx = (80 - 30)/1 = 50 \text{ s}^{-1}$$
$$V_{\text{mid}} = (80 + 30)/2 = 55 \text{ ft/s}$$
$$= (55 \text{ ft/s})(50 \text{ ft/s/ft}) = 2,750 \text{ ft/s}^2$$

Finally

$$dp/dx = (-1.94 \text{ slug/ft}^3)(2,750 \text{ ft/s}^2)$$
$$\boxed{dp/dx = -5,335 \text{ psf/ft}}$$

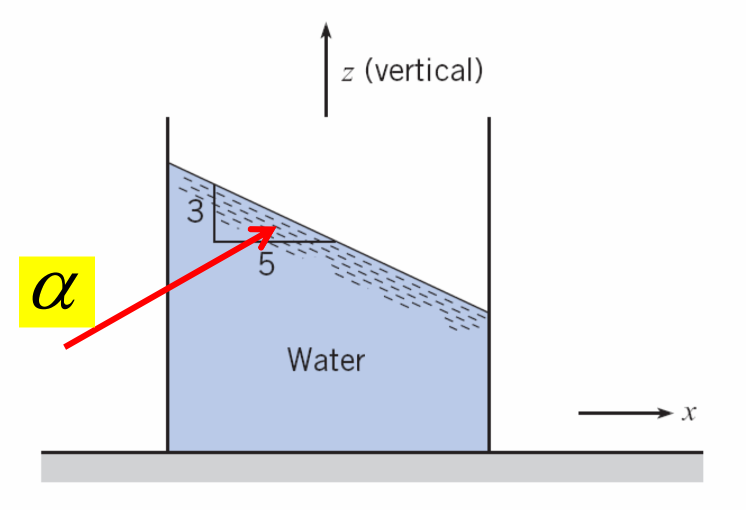
Problem 4.37 (p. 130)

$$-\frac{d}{dl}(p+\gamma z) = \rho a_l$$

$$\frac{dp}{dl} = 0 \text{ as the change in pressure is zero}$$

$$-\frac{d}{dl}(\gamma z) = \rho a_l$$

$$\frac{dz}{dl} = \frac{a_l}{g} = \frac{a_x \cos\alpha}{g} = \sin\alpha$$



$$a_x = g\tan\alpha$$

PROBLEM 4.37

Situation: Tank accelerated in x-direction to maintain liquid surface slope at -5/3.

Find: Acceleration of tank.

APPROACH

Apply Euler's equation.

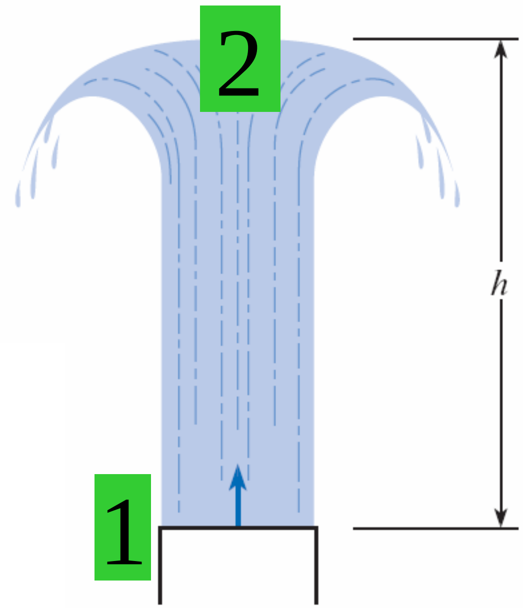
ANALYSIS

Euler's equation. The slope of a free surface in an accelerated tank.

$$\begin{aligned} \tan\alpha &= a_x/g \\ a_x &= g\tan\alpha \\ &= 9.81 \times 3/5 \end{aligned}$$

$$\boxed{a_x = 5.89 \text{ m/s}^2}$$

Problem 4.44 (p. 130)



PROBLEM 4.44

Situation: A water jet is described in the problem statement.

Find: Height h jet will rise.

$Given: V_1 = 20\, m/s$

APPROACH

Apply the Bernoulli equation from the nozzle to the top of the jet. Let point 1 be in the jet at the nozzle and point 2 at the top.

ANALYSIS

Bernoulli equation

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2$$

where $p_1 = p_2 = 0$ gage

$$\begin{aligned} V_1 &= 20 \text{ ft/s} \\ V_2 &= 0 \\ 0 + (20)^2/2g + z_1 &= 0 + 0 + z_2 \\ z_2 - z_1 &= h = 400/64.4 \\ & \boxed{h = 6.21 \text{ ft}} \end{aligned}$$

Problem 4.47 (p. 130)

$$\left(p_1 + \gamma z_1 + \rho \frac{V_1^{\,2}}{2}\right) = \left(p_2 + \gamma z_2 + \rho \frac{V_2^{\,2}}{2}\right)$$

$$z_1 = z_2$$

$$\left(p_1 + \rho \frac{V_1^{\,2}}{2}\right) = \left(p_2 + \rho \frac{V_2^{\,2}}{2}\right)$$

Since $V_2 = 0$

$$\left(\frac{V_1^{\,2}}{2}\right) = \frac{2}{\rho}(p_2 - p_1)$$

$$p_2 = \gamma(l + d) \quad and \quad p_1 = \gamma d$$

$$V_1 = \sqrt{2gl}$$

PROBLEM 4.47

Situation: A glass tube with 90° bend inserted into a stream of water. Water in tube rises 10 inches above water surface.

Find: Velocity.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Hydrostatic equation (between stagnation point and water surface in tube)

$$\frac{p_s}{\gamma} = h + d$$

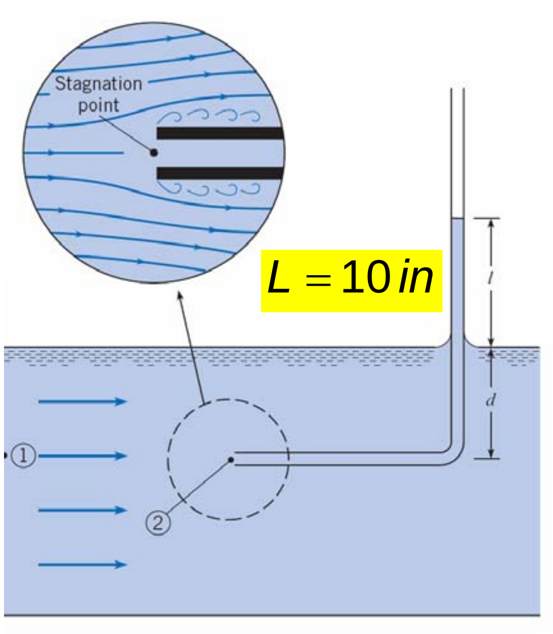
where d is depth below surface and h is distance above water surface.

Bernoulli equation (between freestream and stagnation point)

$$\begin{aligned} \frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\ h + d &= d + \frac{V^2}{2g} \\ \frac{V^2}{2g} &= h \end{aligned}$$

$$V = (2 \times 32.2 \times 10/12)^{1/2}$$

$$\boxed{V = 7.33 \text{ fps}}$$



$$V_1 = ?$$

Problem 4.51 (p. 131)

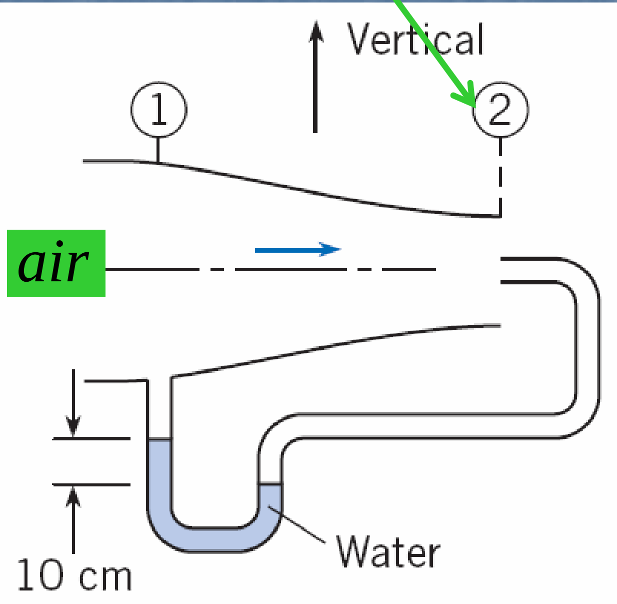
PROBLEM 4.51

Situation: A flow-metering device is described in the problem. Air has density of 1.2 kg/m^3 and a 10 cm deflection of water measured on manometer.

Find: Velocity at station 2. $V_2 = 2V_1$

$V_2 = ?$

Note : (2) *is a stagnation point*



APPROACH

Apply the Bernoulli equation and the manometer equation.

ANALYSIS

Bernoulli equation

$$p_1/\gamma + V_1^2/2g = p_2/\gamma + V_2^2/2g = p_t/\gamma$$

Manometer equation

$$\begin{aligned} p_1 + 0.1 \times 9810 - \overbrace{0.1 \times 1.2 \times 9.81}^{\text{neglect}} &= p_t \\ p_t - p_1 &= 981 \text{ N/m}^2 = \rho V_1^2/2 \\ V_1^2 &= 2 \times 981/1.2 \\ V_1 &= 40.4 \text{ m/s} \\ V_2 &= 2V_1 \\ &\boxed{V_2 = 80.8 \text{ m/s}} \end{aligned}$$

$$V_2 = \left[\frac{2}{\rho}\left(p_{z,1} - p_{z,2}\right)\right]^{1/2}$$

Problem 4.65 (p. 132)

PROBLEM 4.65

Situation: A spherical probe with pressure coefficients given is used to find gas velocity. The pressure difference is 4 kPa and the gas density is 1.5 kg/m^3.

Find: Gas velocity. *Given* $:(C_P)_A = 1.0, (C_P)_B = -0.4$

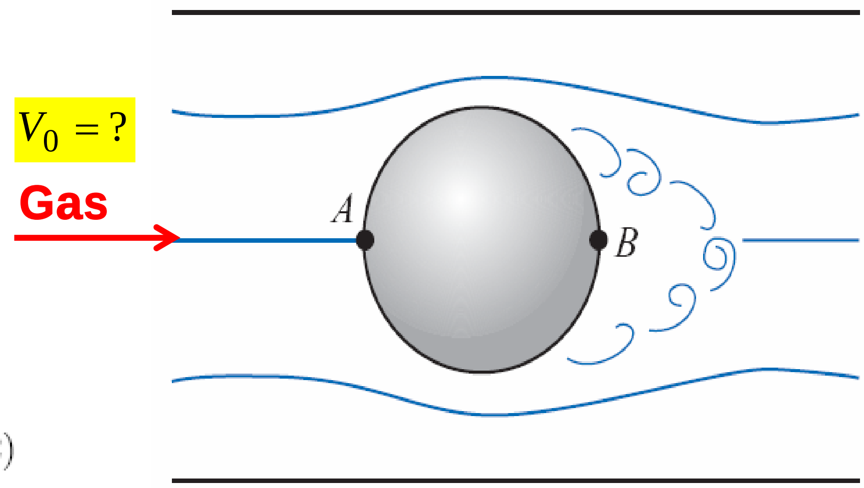
APPROACH

Apply the definition of pressure coefficient.

ANALYSIS

Pressure coefficient

$$\Delta C_p = 1-(-0.4)$$
$$\Delta C_p = 1.4 = (p_A - p_B)/(\rho V_0^2/2)$$
$$V_0^2 = 2(4,000)/(1.5\times 1.4)$$
$$\boxed{V_0 = 61.7 \text{ m/s}}$$



Problem 4.80(p. 135)

PROBLEM 4.80

Situation: Closed tank 4 feet in diameter with piezometer attached is rotated at 15 rad/s about a vertical axis.

Find: Pressure at bottom center of tank.

APPROACH

Apply the equation for pressure variation equation- rotating flow.

ANALYSIS

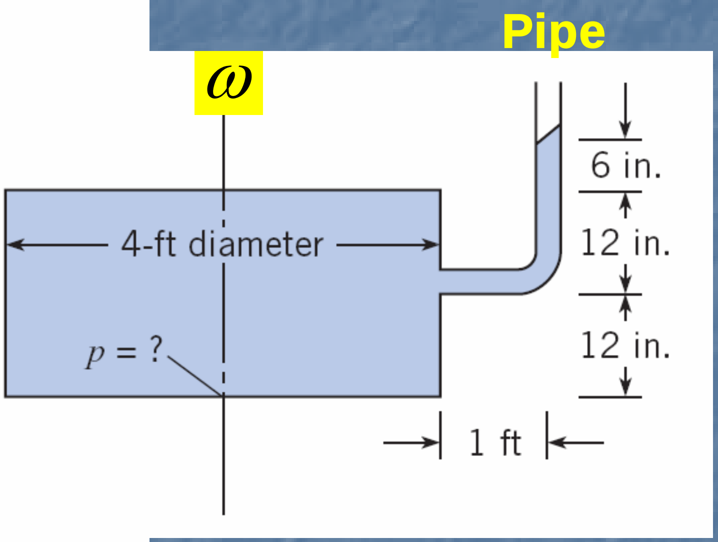
Pressure variation equation- rotating flow

$$p + \gamma z - \rho r^2\omega^2/2 = p_p + \gamma z_p - \rho r_p^2\omega^2/2$$

where $p_p = 0$, $r_p = 3$ ft and $r = 0$, then

$$\begin{aligned} p &= -(\rho/2)(9 \times 225) + \gamma(z_p - z) \\ &= (1.94/2)(2025) + 62.4 \times 2.5 \\ &= -1808 \text{ psfg} = -12.56 \text{ psig} \end{aligned}$$

$$\boxed{p = -12.6 \text{ psig}}$$



Problem 4.83(p. 135)

PROBLEM 4.83

Situation: A U-tube rotating about one leg. Before rotation, the level of liquid in each leg is 0.25 m. The length of base and length of leg is 0.5 m.

Find: Maximum rotational speed so that no liquid escapes from the leg.

APPROACH

Apply the pressure variation equation for rotating flow. Let point 1 be at top of outside leg and point 2 be at surface of liquid of inside leg.

ANALYSIS

At the condition of imminent spilling, the liquid will be to the top of the outside leg and the leg on the axis of rotation will have the liquid surface at the bottom of its leg.

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

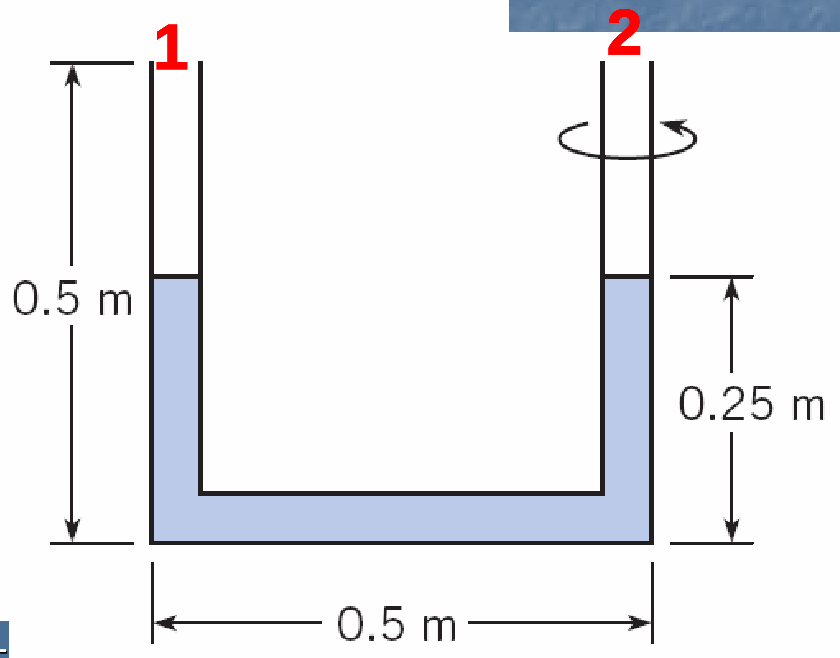
where $p_1 = p_2$, $z_1 = .5$ m and $z_2 = 0$

$$\gamma \times 0.5 - (\gamma/g) \times .5^2 \omega^2/2 = 0$$

$$\omega^2 = 4g$$

$$= 2\sqrt{g}$$

$$\boxed{\omega = 6.26 \text{ rad/s}}$$



Problem 4.88(p. 136)

PROBLEM 4.88

Situation: A manometer is rotated about one leg. There is a 25 cm height difference in liquid (S=0.8) between the legs. The length of the base is 10 cm.

Find: Acceleration in g's in leg with greatest amount of oil..

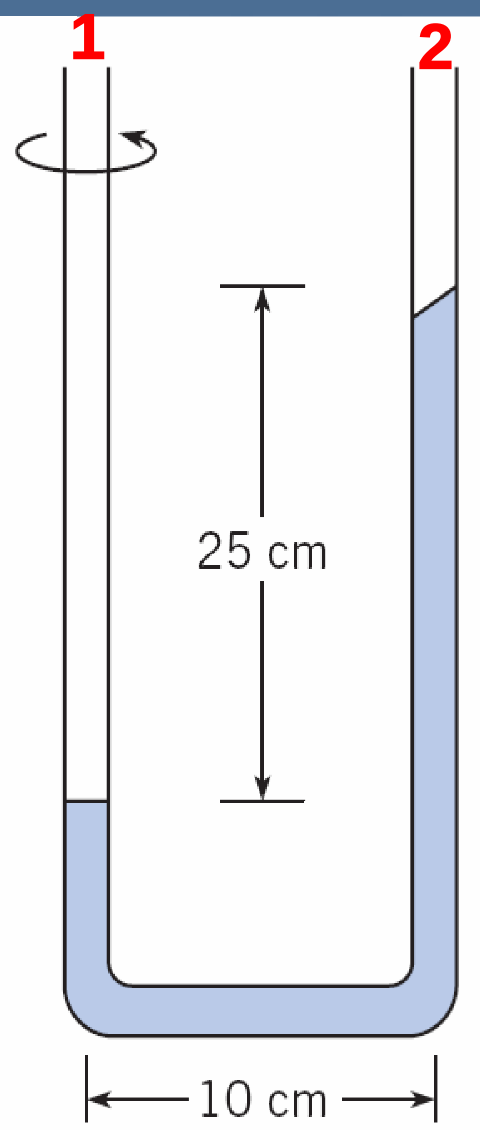
APPROACH

Apply the pressure variation equation for rotating flow between the liquid surfaces of 1 & 2Let leg 1 be the leg on the axis of rotation. Let leg 2 be the other leg of the manometer.

ANALYSIS

Pressure variation equation- rotating flow

$$\begin{aligned} p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 &= p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2 \\ 0 + \gamma z_1 - 0 &= \gamma z_2 - (\gamma/g) r_2^2 \omega^2/2 \\ \omega^2 r_2^2/(2\text{g}) &= z_2 - z_1 \\ a_n &= r\omega^2 \\ &= (z_2 - z_1)(2\text{g})/r \\ &= (0.25)(2\text{g})/r_2 \\ &= (0.25)(2\text{g})/0.1 \\ & \boxed{a_n = 5g} \end{aligned}$$



Problem 4.93(p. 137)

PROBLEM 4.93

Situation: Mercury in rotating manometer with dimensions shown on figure.

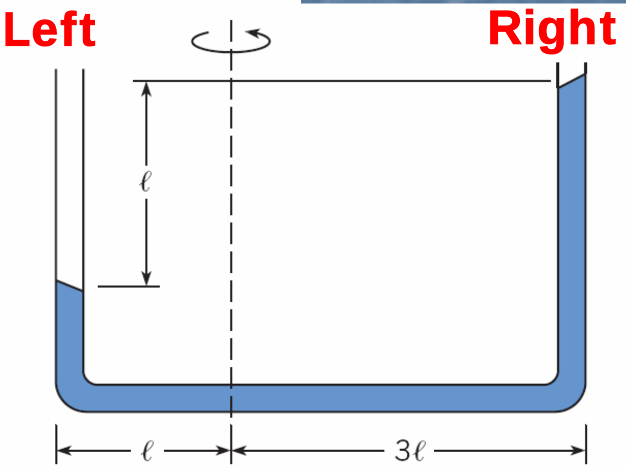
Find: Rate of rotation in terms of g and ℓ.

APPROACH

Apply the pressure variation equation for rotating flow from the mercury surface in the left tube to the mercury surface in the right tube. Then $p_\ell = p_r$.

ANALYSIS

Pressure variation equation- rotating flow



$$
\begin{aligned}
\gamma z_\ell - \rho r_\ell^2 \omega^2/2 &= \gamma z_r - \rho r_r^2 \omega^2/2 \\
\omega^2(\gamma/2g)(r_r^2 - r_\ell^2) &= \gamma(z_r - z_\ell) \\
\omega^2 &= 2g(z_r - z_\ell)/(r_r^2 - r_\ell^2) \\
&= 2g(\ell)/(9\ell^2 - \ell^2)
\end{aligned}
$$

$$\boxed{\omega = \sqrt{g/(4\ell)}}$$

Problem 4.103(p. 138)

PROBLEM 4.103

Situation: The velocity at outlet pipe from a reservoir is 6 m/s and reservoir height is 15 m.

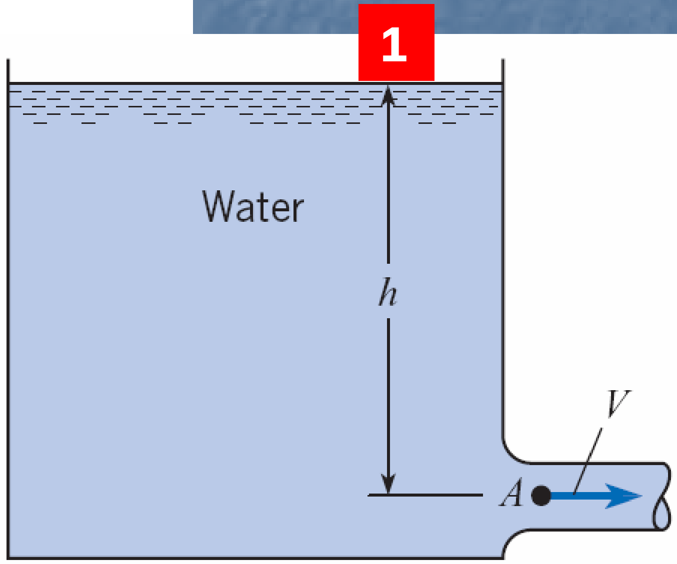
Find: Pressure at point A.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation Let point 1 be at reservoir surface.



$$
\begin{aligned}
(p_1/\gamma) + (V_1^2/2g) + z_1 &= (p_A/\gamma) + (V_A^2/2g) + z_A \\
0 + 0 + 15 &= p_A/9810 + 6^2/(2 \times 9.81) + 0 \\
p_A &= (15 - 1.83) \times 9810 \\
p_A &= 129,200 \text{ Pa, gage}
\end{aligned}
$$

$$\boxed{p_A = 129.2 \text{ kPa, gage}}$$

THE END